Extremal, enumerative and probabilistic results on ordered hypergraph matchings

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Joint work with Michael Anastos, Matthew Kwan and Benny Sudakov

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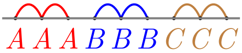

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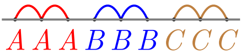

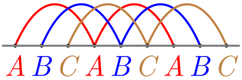

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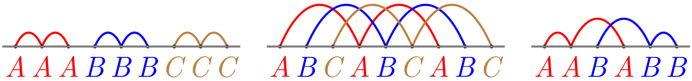

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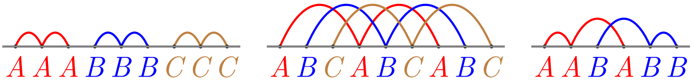

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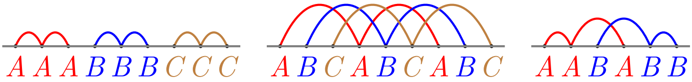

- For an r-matching of size n, we can write it as a string on an alphabet of size n: AAABBBCCC, ABCABCABC, AABABB.

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  - Say AAABBBCCCDDD is a AAABBB-clique of size 4.

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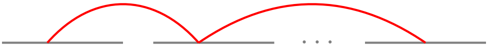

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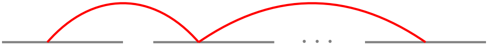

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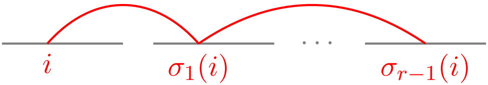

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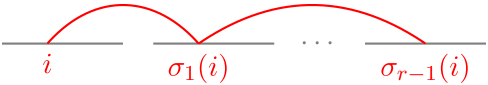

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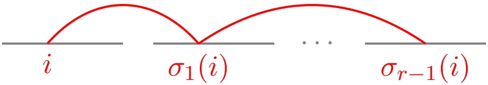

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- Note: $\#(r\text{-partite } r\text{-patterns}) = 2^{r-1}$.

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- Initiated by Dudek, Grytczuk and Ruciński.

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- Takeaway: r-patterns are **not** "independent"; many of them contribute parallelly.

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- The former part is via a subadditivity argument with concentration inequalities.

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- See Dudek–Grytczuk–Przybyło–Ruciński for progress and cute conjectures.

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Fix an r-partite r-pattern P. How many edges can an ordered r-uniform hypergraph have with no P-clique of size $m+1$?

Theorem (Anastos–J.–Kwan–Sudakov 2025)

It is at least $\binom{n}{r} - \binom{n-rm}{r} \approx rm\binom{n}{r-1}$ *and at most* $O\left(r^2 m\binom{n}{r-1}\right)$.

Conjecture (Anastos–J.–Kwan–Sudakov 2025)

For every r-partite r-pattern P, the answer is $\binom{n}{r} - \binom{n-rm}{r}$.

- We verified the conjecture for the cases when
  - $m = 1$ or

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  - $P = |AB|BA|AB|BA|\ldots$ is the alternating r-pattern.

The End

Questions? Comments?